HOW TO EXPLAIN THE EFFICIENCY OF TRIANGULAR AND TRAPEZOID MEMBERSHIP FUNCTIONS IN APPLICATIONS TO DESIGN

A. Gholamy[1], O. Kosheleva[2], V. Kreinovich[3]

[1] *Department of Geological Sciences, University of Texas at El Paso, El Paso, Texas. 79968, USA*
afshingholamy@gmail.com

[2] *Department of Teacher Education, University of Texas at El Paso, El Paso, Texas. 79968, USA*
olgak@utep.edu

[3] *Department of Computer Science, University of Texas at El Paso, El Paso, Texas. 79968, USA*
vladik@utep.edu

Abstract

It is well known that expert knowledge is very important for solving design problems. However, expert knowledge is not easy to describe in precise terms, since experts often use imprecise ("fuzzy") words from natural language such as "small" or "large". In order to describe such knowledge in precise terms – which would be understandable to a computer – Lotfi Zadeh came up with a special methodology that he called *fuzzy*. This methodology had many successful applications, in particular, applications to design. The first stage of the general fuzzy methodology is eliciting, from the expert, a *membership function* corresponding to each imprecise term, i.e., a function that assigns, to each possible value of the corresponding quantity, a degree to which this value satisfies this property (e.g., a degree to which, in the expert's opinion, this given value is small). If we follow the expert's opinion very closely, we often come up with very complex membership functions. However, surprisingly, in many applications, the simplest membership functions - of triangular or trapezoid shape - turned out to be more efficient than the supposedly more adequate complex ones. This is counterintuitive: the closer we follow the expert's opinion, the worse our result. Some explanations for this seemingly counterintuitive phenomenon have been proposed earlier. However, these explanations only work when we use the simplest possible "and"-operation – minimum, while this phenomenon has been observed for other "and"-operations as well. In this paper, we provide a new, more general explanation for the above phenomenon, an explanation that works for all possible "and"-operations.

Key words: expert knowledge, fuzzy methodology, complicated membership functions, trapezoid membership functions, triangular membership functions.

Introduction

For engineering design, expert knowledge is often important. In many engineering design problems, it is often very important to take into account the opinion of the human experts.

Expert knowledge is not easy to take into account. Taking into account expert's knowledge is not always easy, since experts often formulate their knowledge not in precise computer-understandable terms, but by using imprecise (fuzzy) words from natural language such as "small", "approximately", "close", etc. To take imprecise expert knowledge into account, Lotfi Zadeh came up with special fuzzy techniques [1-6].

1   First stage of fuzzy methodology: eliciting membership functions.

In the fuzzy approach, first, we describe the meaning of the corresponding natural-language words in precise numerical terms. For this purpose, for each such word (e.g., for the word "small"):

- We provide the expert with several possible values x of the corresponding quantity,
- For each of these values, we ask the expert to mark a point, on a scale from 0 to 1, to what extent the given value satisfies this property (e.g., to what extent this value is small).

The resulting points $\mu_P(x) \in [0,1]$ corresponding to different values x form what is called a membership function or a fuzzy set describing the corresponding natural-language property P.

2 Second stage of fuzzy methodology: combining membership degrees

An additional problem stems from the fact that many expert rules have several conditions: e.g., "if the road will have heavy traffic and the region experiences drastic changes from freezing to thawing, then the road pavement must be made reasonably resilient to such changes." The condition to this rule contains two imprecise natural-language words: "heavy" and "drastic". In the ideal world,

- In addition to asking the experts for all possible values of $\mu_{heavy}(t)$ and $\mu_{drastic}(c)$,
- we should also ask, for all possible combinations of traffic t and temperature change c, to what extent the above condition is satisfied for the corresponding pair (t, c).

However, in practice, already asking the expert about all possible values of one quantity takes a long time. So, asking the expert about all possible pairs is not realistically possible.

Since we cannot elicit, from the expert, the degree to which each pair satisfies the corresponding condition, we must therefore estimate this degree based on what we know, i.e., on the degrees to which the traffic is heavy and to which the temperature change is drastic. In general, we know the degrees a and b to which imprecise statements A and B are true, and based on these degrees, we need to estimate the degree to which the "and"-combination $A \& B$ is true. Let us denote the corresponding estimate by $f_\&(a, b)$.

The algorithm that computes these estimates based on the two given degrees is known as an "and"-operation or a t-norm. The t-norm has to satisfy several reasonable properties. For example, since $A \& B$ means the same as $B \& A$, we expect that the estimates for these two "and"-combinations should be the same, i.e., that $f_\&(a, b) = f_\&(b, a)$ for all a and b.

Similarly, since $A \& (B \& C)$ and $(A \& B) \& C$ mean the same, we expect that the resulting estimates are equal, i.e., that $f_\&(a, f_\&(b, c)) = f_\&(f_\&(a, b), c)$ for all a, b, and c.

All t-norms that satisfy all these properties are known. The most widely used "and"-operations are $f_\&(a, b) = \min(a, b)$ and $f_\&(a, b) = a \cdot b$, but many other operations are also used.

3 Triangular and trapezoid membership functions are usually very efficient

In principle, if we ask an expert, we can get different shapes of membership functions. In the beginning, practitioners tried to describe these membership functions as accurately as possible. However, it soon turned out that in most applications, it is sufficient to consider simple membership functions whose graphs have triangular or trapezoid shape (See Figure 1).

To be more precise, triangular membership functions are the following functions which are different from 0 on an interval $[\underline{x}, \overline{x}]$ with midpoint $\tilde{x}$:

- they linearly increase from 0 to 1 when x is smaller than the midpoint $\tilde{x}$ and then
- they linearly decrease from 1 to 0 when x is larger than the midpoint $\tilde{x}$.
- In precise terms:
  - $\mu(x) = \frac{x - \underline{x}}{\tilde{x} - \underline{x}}$ when $\underline{x} \leq x \leq \tilde{x}$
  - $\mu(x) = \frac{\overline{x} - x}{\overline{x} - \tilde{x}}$ when $\tilde{x} \leq x \leq \overline{x}$
  - $\mu(x) = 0$ when $x < \underline{x}$ and or when $x > \overline{x}$.

Trapezoid membership functions also have an interval in the middle when the function is identically equal to 1. To be more precise, we select four values $\underline{x} < \underline{t} < \overline{t} < \overline{x}$ and then we take:

- $\mu(x) = 0$ when $x \leq \underline{x}$
- $\mu(x) = \dfrac{x - \underline{x}}{\underline{t} - \underline{x}}$ when $\underline{x} \leq x \leq \underline{t}$
- $\mu(x) = 1$ when $\underline{t} \leq x \leq \overline{t}$
- $\mu(x) = \dfrac{\overline{x} - x}{\overline{x} - \overline{t}}$ when $\overline{t} \leq x \leq \overline{x}$
- $\mu(x) = 0$ when $x \geq \overline{x}$

Open problem: why are these membership functions efficient? While empirical evidence shows that triangular and trapezoid membership functions are efficient in many engineering applications, there is still no convincing general explanation for this empirical efficiency.

A partial explanation was provided in [7], but this explanation is only valid when we use a minimum t-norm. In this paper, we show that a similar explanation can be made general by extending it to general "and"-operations.
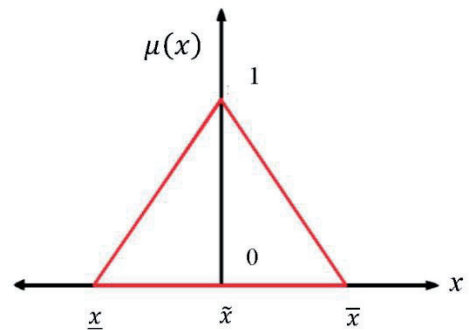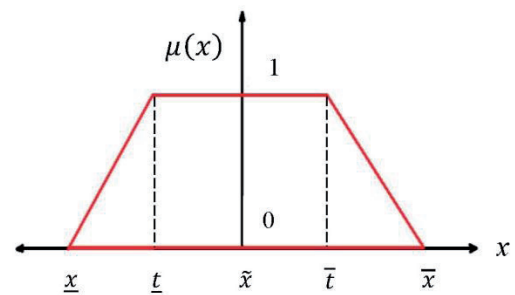

Figure1 - Graphs of membership functions have triangular or trapezoid shape

4 Main idea behind known partial explanation: a brief reminder

To explain this main idea, let us recall, in some detail how membership functions are usually elicited.

Elicitation of membership functions: first step. Usually, for each property, there is a threshold $\underline{x}$ below which, according to the expert, the corresponding property is definitely not satisfied. For example, experts may have different opinions on what constitutes warm, but for every expert, there is a temperature below which it is clearly not warm. For some people from the North, this threshold may be 14 C, for people from the tropics, even 19 C may be chilly, so for them it will be around 22 C, but there is such threshold for every expert.

In precise terms, this means that $\mu(x) = 0$ when $x \leq \underline{x}$.

Elicitation of membership functions: second step. Similarly, there exists a threshold $\overline{x}$ above which the corresponding property is definitely not satisfied. This means that $\mu(x) = 0$ when $x \geq \overline{x}$.

Elicitation of membership functions: third step. There also usually exist values for which the corresponding property is definitely satisfied. For example, for "warm", most people will agree that 25 C is warm.

Sometimes, there is a single value $\tilde{x}$ for which the original property is definitely satisfied. Sometimes, there is a whole interval of values $[\underline{t}, \overline{t}]$ for all of which the given property is definitely satisfied. For all such values x, we have $\mu(x) = 1$.

Elicitation of membership functions: fourth step. For the intervals $[\underline{x}, \tilde{x}]$ and $[\tilde{x}, \overline{x}]$ (or, alternatively, $[\underline{x}, \underline{t}]$ and, $[\overline{t}, \overline{x}]$), the membership degrees change from 0 to 1 and then from 1 to 0. These are the values that we need to elicit from the expert.

For each of the intervals, to elicit these values, we provide the expert with several values
$$x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n$$

from the corresponding interval - where x_0 and x_n are the interval's endpoints - and elicit the corresponding membership degrees $\mu_i \stackrel{\text{def}}{=} \mu(x_i)$ for $i = 1, \ldots, x_n$ (the values μ_0 and μ_n corresponding to endpoints are known). Usually, the values x_i are equally spaced: $x_1 - x_0 = x_2 - x_1 = \cdots$, so that $\mu_i = \mu_0 + i \cdot h$, where we denoted $h \stackrel{\text{def}}{=} x_1 - x_0$.

Main idea behind the known (partial) explanation of the effectiveness of triangular and trapezoid membership functions. When the value x' is close to the value x ($x' \approx x$), we do not expect that the expert's degree of confidence $\mu(x')$ that x' satisfies the given property to be much different from the expert's degree of confidence $\mu(x)$ that the original value x satisfies this property: we should have $\mu(x) \approx \mu(x')$. For example: **if** x is reasonably small, and x' is close to x, **then** it is reasonable to conclude that x' should also be reasonably small - with almost the same degree of smallness as x.

In other words, if x and x' are close, then the values $\mu(x)$ and $\mu(x')$ should also be close. For large n, the differences between x_{i-1} and x_i are small, so x_{i-1} is close to x_i. Thus, we conclude that for every i, the values μ_{i-1} and μ_i should be close.

The more this property is satisfied, the more it is reasonable that the values μ_i adequately describe the expert's knowledge. It thus makes sense to select the values μ_i that satisfy the above property to the largest possible degree. In precise terms: **for** each possible sequence of the values μ_i, we can find the degree to which the above property is satisfied, and **then** as the most adequate description of the expert's knowledge, we select the values μ_i for which this degree is the largest possible.

How can we describe this degree? What does it mean that two values μ_{i-1} and μ_i are close? Intuitively, it means that the absolute value of their difference $|\mu_i - \mu_{i-1}|$ is small. Let $s(v)$ denote the membership function corresponding to "small". Then, for each i the degree to which μ_i and μ_{i-1} are close is equal to $s(|\mu_i - \mu_{i-1}|)$, and the degree to which this closeness condition is satisfied for $i = 1$ and for $i = 2$, etc., is equal to

(1) $$f_\&(s(|\mu_1 - \mu_0|), s(|\mu_2 - \mu_1|), \ldots, s(|\mu_n - \mu_{n-1}|))$$

for an appropriate "and"-operation $f_\&(a, b)$.

We must find the values $\mu_1, \ldots, \mu_{n-1}$ for which the expression (1) attains the largest possible value.

5   Analysis of the problem and the resulting justification

It is reasonable to expect that this optimization problem has a unique solution. In engineering problems, in principle, we may have optimality criteria that allow many different solutions. For the same problem, we have several solutions which are equally good according to the selected criterion. However, from the practical viewpoint, this means that the corresponding criterion is not final.

For example, if we have several plane designs with similar energy consumption and similar manufacturing and exploitation costs, this means that we can select, among these designs, the one with the smallest negative effect on the environment - and thus narrow down the set of all optimal designs. Eventually, we will end up with a final optimality criterion for which exactly one alternative is optimal. From this viewpoint, it is reasonable to require that our criterion (1) is, in this sense, final - i.e., that it leads to the unique selection of the corresponding membership degrees μ_i.

Let us describe this requirement in precise terms.

Definition 1

- *By a* membership function, *we will (as usual) mean a mapping* $s(v)$ *from real numbers to the interval* [0,1].
- *By an n-aggregation operation, we mean a function* $f(a_1, \ldots, a_n)$ *of n variables* $a_i \in [0,1]$ *with values from* [0,1] *which is symmetric, i.e., for which:*

$$f(a_1, \ldots, a_n) = f\left(a_{\pi(1)}, \ldots, a_{\pi(n)}\right)$$

for all a_i and for any permutation π.

- *We say that a pair $\langle f_\&, s \rangle$, where $f_\&$ is an n-aggregation operation and s is a membership function, is* final *for increasing sequences if there exists exactly one sequence of values $\mu_0 = 0 < \mu_1 < \mu_2 < \cdots < \mu_n = 1$ for which the expression (1) attains its largest possible value. The corresponding sequence μ_i will be called the* optimal increasing sequence.

Proposition 1

For every pair $\langle f_\&, s \rangle$ which is final for increasing sequences, the optimal increasing sequence has the form $\mu_i = \frac{i}{n}$.

Comments

- For reader's convenience, the proof is given in the special (last) Proofs section.
- According to this result, for the optimal increasing section of the membership function, we have $\mu(x_i) = \mu(x_0 + i \cdot h) = \mu_i = \frac{i}{n}$. Let us describe $\mu(x_i)$ in terms of x_i.

  From $x_i = x_0 + i \cdot h$, we conclude that $i = \frac{x_i - x_0}{h}$, thus $\mu(x_i) = \frac{x_i - x_0}{n \cdot h}$, i.e., $\mu(x_i) = a \cdot x_i + b$ where $a \stackrel{\text{def}}{=} \frac{1}{n \cdot h}$ and $b \stackrel{\text{def}}{=} -\frac{x_0}{n \cdot h}$. So, the optimal membership function $\mu(x)$ on the interval $[\underline{x}, \tilde{x}]$ is a linear function that takes the value 0 for $x = \underline{x}$, and the value 1 for $x = \tilde{x}$, thus, $\mu(x) = \frac{x - \underline{x}}{\tilde{x} - \underline{x}}$. This is exactly the increasing linear segment that we have been trying to explain.

- Proposition 1 does not require that the *n*-aggregation operation is minimum - it can be any t-norm. Moreover, we do not even require that this operation be a t-norm: we do not require its associativity or monotonicity. So, our result is even more general than what we wanted.
- A similar result holds for optimal decreasing sequences.

Definition 2

We say that a pair $\langle f_\&, s \rangle$, where $f_\&$ is an n-aggregation operation and s is a membership function, is final for decreasing sequences *if there exists exactly one sequence of values $\mu_0 = 1 > \mu_1 > \mu_2 > \cdots > \mu_n = 0$ for which the expression (1) attains its largest possible value. The corresponding sequence μ_i will be called the* optimal decreasing sequence.

Proposition 2

For every pair $\langle f_\&, s \rangle$, which is final for decreasing sequences, the optimal decreasing sequence has the form $\mu_i = 1 - \frac{i}{n}$.

Comment

According to this result, for the optimal decreasing section of the membership function, we have $\mu(x_i) = \mu(x_0 + i \cdot h) = \mu_i = 1 - \frac{i}{n}$. Let us describe $\mu(x_i)$ in terms of x_i. From $x_i = x_0 + i \cdot h$, we have $i = \frac{x_i - x_0}{h}$, thus $\mu(x_i) = 1 - \frac{x_i - x_0}{n \cdot h}$, i.e., $\mu(x_i) = a \cdot x_i + b$, where $a \stackrel{\text{def}}{=} -\frac{1}{n \cdot h}$ and $b \stackrel{\text{def}}{=} 1 + \frac{x_0}{n \cdot h}$.

So, the optimal decreasing membership function $\mu(x)$ on the interval $[\tilde{x}, \overline{x}]$, is a linear function that takes the value 1 for $x = \tilde{x}$, and the value 0 for $x = \overline{x}$, thus, $\mu(x) = \frac{\overline{x} - x}{\overline{x} - \tilde{x}}$. This is exactly the decreasing linear segment that we have been trying to explain.

6 Proof of Propositions 1 and 2

It is sufficient to prove Proposition 1, the proof of Proposition 2 is similar.

Let us prove, by contradiction, that for the optimal increasing sequence μ_i, we have $\mu_1 - \mu_0 = \mu_2 - \mu_1 = \cdots$, i.e., that all the differences $\Delta_i \stackrel{\text{def}}{=} \mu_i - \mu_{i-1}$ are equal to each other.

Indeed, assume that some of these differences are different, i.e., $\Delta_{i'} \neq \Delta_{i''}$ for some i' and i''. Without losing generality, we can assume that $i' < i''$. In this case, we can form the following auxiliary sequence μ_i'.

- for $i < i'$, we take $\mu_i' = \mu_i$;

- for $i = i'$, we take $\mu'_{i'} = \mu_{i'-1} + \Delta_{i''}$;
- when $i' < i < i''$, we take $\mu'_i = \mu_i + (\Delta_{i''} - \Delta_{i'})$;
- finally, for $i \geq i''$, we again take $\mu'_i = \mu_i$.

This sequence is different from the original sequence μ_i, since $\Delta_{i'} \neq \Delta_{i''}$ and thus,

$$\mu'_{i'} = \mu_{i'-1} + \Delta_{i''} \neq \mu'_{i'} = \mu_{i'-1} + \Delta_{i'} = \mu_{i'}.$$

On the other hand, the differences $\Delta'_i \stackrel{\text{def}}{=} \mu'_i - \mu'_{i-1}$ corresponding to the new sequence has the following form:

- for $i < i'$, we have $\Delta'_i = \Delta_i$,
- for $i = i'$, we have $\Delta'_{i'} = \Delta_{i''}$;
- when $i' < i < i''$, we get $\Delta'_i = \Delta_i$;
- for $i = i''$, we get $\Delta'_{i''} = \Delta_{i''}$; and
- for $i > i''$, we again get $\Delta'_i = \Delta_i$.

Thus, the new sequence of differences Δ'_i is obtained from the original sequence of differences Δ_i by an appropriate permutation π: namely, by a permutation that swaps the indices i' and i'' and leaves all the other indices unchanged. Thus, the values $s(\Delta'_i) = s(|\mu'_i - \mu'_{i-1}|)$ are also obtained from the values $s(\Delta_i) = s(|\mu_i - \mu_{i-1}|)$ by a similar permutation.

Since we assumed that the n-aggregation operation is symmetric, i.e., that the result of applying this operation does not change if we simply permute the inputs, we thus conclude that

$$(2) \qquad f_\&\big(s(|\mu'_1 - \mu'_0|), s(|\mu'_2 - \mu'_1|), \ldots, s(|\mu'_n - \mu'_{n-1}|)\big)$$
$$= f_\&\big(s(|\mu_1 - \mu_0|), s(|\mu_2 - \mu_1|), \ldots, s(|\mu_n - \mu_{n-1}|)\big)$$

i.e., that the expression (1) attains the same value for both sequences μ_i and μ'_i.

Since we assumed that the sequence μ_i is optimal, this means that for this sequence, the value of the expression (1) is the largest possible. Thus, the above equality (2) shows that the value of the expression (1) for the new sequence μ'_i is also optimal. So, we have two different sequences μ_i and μ'_i on which the expression (1) attains its maximum – which contradicts to our assumption that the pair $\langle f_\&, s \rangle$ is final for increasing sequences.

This contradiction shows that the differences Δ_i cannot be different, so they are all equal:

$$\Delta_1 = \Delta_2 = \cdots = \Delta_n, \text{ i.e., } \mu_1 - \mu_0 = \mu_2 - \mu_1 = \cdots = \Delta_1$$

Thus:

- From $\mu_0 = 0$, we conclude that $\mu_1 = \Delta_1$.
- From $\mu_2 - \mu_1 = \Delta_1$, we conclude that $\mu_2 = \mu_1 + \Delta_1 = 2\Delta_1$.
- By induction, once we have shown that $\mu_i = i \cdot \Delta_1$, we can conclude that $\mu_{i+1} = \mu_i + i \cdot \Delta_1 + \Delta_1$ and thus, that $\mu_{i+1} = (i+1) \cdot \Delta_1$.
- So, we conclude that $\mu_i = i \cdot \Delta_1$ for all i.
- In particular, for $i = n$, we conclude that $\mu_n = n \cdot \Delta_1$, hence $\Delta_1 = \frac{1}{n}$ and thus,

$$\mu_i = i \cdot \Delta_1 = \frac{i}{n}.$$

The proposition is proven.

Acknowledgments

References

[1] **Zadeh LA.** Fuzzy sets. *Information and Control.* - 1965; 8: 338–353.

[2] **Klir GJ, Yuan Bo**. Fuzzy Sets and Fuzzy Logic, Theory and Applications. Prentice Hall, Upper Saddle River, New Jersey. 1995. – 592 p.

[3] **Novak V, Perfilieva I, Mockor J**. Mathematical Principles of Fuzzy Logic. Kluwer, Boston, Dordrecht. 1999. (Математические принципы нечёткой логики / Новак В., Перфильева И., Мочкор Ж. //Пер с англ.; Под ред. Аверкина А.Н. – М.: ФИЗМАТЛИТ, 2006. – 352 с.)

[4] **Nguyen HT, Walker EA**. A First Course in Fuzzy Logic. Chapman and Hall/CRC, Boca Raton, Florida. 2006. - 430 p.

[5] **Belohlavek R, Dauben JW, Klir GJ**. Fuzzy Logic and Mathematics: A Historical Perspective. Oxford University Press, New York. 2017. – 522 p. - https://b-ok.org/book/2925381/efa85d.

[6] **Mendel JM**. Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions. Springer, Cham, Switzerland. 2017. - https://doi.org/10.1007/978-3-319-51370-6.

[7] **Kosheleva O, Kreinovich V, Shahbazova S**. Type-2 Fuzzy Analysis Explains Ubiquity of Triangular and Trapezoid Membership Functions, Proceedings of the World Conference on Soft Computing, Baku, Azerbaijan, May 29–31. 2018.

КАК ОБЪЯСНИТЬ ЭФФЕКТИВНОСТЬ ТРЕУГОЛЬНЫХ И ТРАПЕЦИЕВИДНЫХ ФУНКЦИЙ ПРИНАДЛЕЖНОСТИ В ПРИЛОЖЕНИЯХ К ПРОЕКТИРОВАНИЮ

А. Голами[1], О. Кошелева[2], В. Крейнович[3]

[1] *Кафедра геологических наук, Техасский университет – Эль Пасо, Эль Пасо, Техас, 79968, США,*
afshingholamy@gmail.com
[2] *Кафедра подготовки учителей, Техасский университет – Эль Пасо, Эль Пасо, Техас, 79968, США,*
olgak@utep.edu
[3] *Кафедра информатики, Техасский университет – Эль Пасо, Эль Пасо, Техас, 79968, США,*
vladik@utep.edu

Аннотация

Хорошо известно, что знания экспертов очень важны для решения задач проектирования. Однако эти знания не так просто описать в чётких терминах, так как эксперты часто используют нечёткие слова, такие как „небольшой" или „большой". Чтобы описывать такого типа знание в чётких терминах – терминах понятных компьютеру - Лотфи Задэ придумал специальную методологию, которую он назвал методологией нечётких множеств. Эта методология имеет много приложений, в частности в проектировании. Первая стадия этой методологии состоит в том, чтобы „извлечь" из экспертов функции принадлежности соответствующие различным нечётким терминам. Такая функция сопоставляет каждому возможному значению соответствующей физической величины число, выражающее - до какой степени это значение удовлетворяет данному свойству (например, до какой степени можно сказать, что это значение небольшое). Если попытаться очень точно отразить мнение эксперта, то часто получают очень сложную функцию принадлежности. Однако на практике гораздо лучшие результаты получаются, когда используются простейшие функции принадлежности: треугольные и трапециевидные. На первый взгляд, это противоречит нашей интуиции: чем точнее описывают мнение эксперта, тем хуже получается результат. Исследователи придумали объяснение этого странного феномена, но это объяснение работает только когда для описания логической „и"-операции используется минимум, в то время как этот феномен наблюдается и когда используются другие „и"-операции. В настоящей статье приводится новое более общее объяснение, которое применимо для всех возможных „и"-операций.

Ключевые слова: экспертные знания, методология нечётких множеств, сложные функции принадлежности, трапециевидные функции принадлежности, треугольные функции принадлежности.

Благодарности

Сведения об авторах

Афшин Голами (1979 г.р.) получил диплом магистра по геофизике полезных ископаемых, со специализацией по сейсмологии Тегеранского университета в Иране в 2010 г. Второй диплом магистра и диплом Доктора философии по вычислительной геофизике он получил в Техасском университете Эль Пасо в 2014 и 2018 гг. Его научные результаты включают вейвлетный анализ сейсмических сигналов. Он активно участвовал в междисциплинарном проекте по оптимизации процесса уплотнения дорожного покрытия.

Afshin Gholamy (b 1979) received his MS in exploration geophysics with concentration in seismology from University of Tehran, Iran in 2010, his second MS and PhD in computational geophysics from the University of Texas at El Paso, USA in 2014 and 2018. During his doctoral studies, he worked on wavelet analysis of seismic signals and provided theoretical proof to the success of Ricker wavelets in seismic data processing. He also participated in an interdisciplinary research effort to study and optimize intelligent compaction of soil system.

Ольга Кошелева (1956 г.р.) получила диплом математика Новосибирского университета в 1978 г., диплом магистра по информатике в Техасском университете в Эль Пасо в 1994 г. и диплом Доктора философии по вычислительной технике в этом же университете в 2003 г. С 1990 г. она работает в Техасском университете в Эль Пасо. Она работала приглашенным исследователем в Эйлеровском международном математическом институте в С. Петербурге (2002) и в Католическом университете в Пелотас, Бразилия (2003-2004). Она опубликовала научную монографию, более 50 глав в научных сборниках, более 220 научных статей в журналах и более 190 статей в реферируемых трудах конференций.

Olga Kosheleva (b 1956) received her M.Sc. in Mathematics from Novosibirsk University, Russia, in 1978, and M.Sc. in Computer Science (1994) and Ph.D. in Computer Engineering (2003) from the University of Texas at El Paso. Since 1990, she is with the University of Texas at El Paso. She also served as a Visiting Researcher with the Euler International Mathematical Institute, St. Petersburg, Russia (2002), and with the Catholic University of Pelotas, Brazil (2003-2004). Main current interest: applications of information technology, especially applications to education in mathematics, science, and engineering. Published a research monograph, more than 50 book chapters, more than 220 journal papers, and more than 190 papers in refereed conference proceedings.

Владик Крейнович (1952 г.р.) получил диплом по математике и информатике в С. Петербургском (Ленинградском) университете в 1974 г. и диплом кандидата наук в Институте математики Сибирского отделения Академии наук в Новосибирске в 1979 г. С 1990 г. работает на кафедре информатики Техасского университета в Эль Пасо. Работал приглашённым исследователем в Париже, Ганновере, Гонконге, С. Петербурге и в Бразилии. Опубликовал семь монографий, более 1400 научных статей, редактировал 20 сборников статей. Владик - член редколлегии журнала «Надёжные вычисления» и других журналов, Вице-президент Международной ассоциации по Нечётким множествам и Вице-президент Европейского общества по нечётной логике и её приложениям; Действительный член Мексиканского общества по Искусственному интеллекту. - http://www.cs.utep.edu/vladik/.

Vladik Kreinovich (b 1952) received his MS in Mathematics and Computer Science from St. Petersburg University, Russia, in 1974, and PhD from the Institute of Mathematics, Soviet Academy of Sciences, Novosibirsk, in 1979. Since 1990, he has worked in the Department of Computer Science at the University of Texas at El Paso. In addition, he has served as an invited professor in Paris, Hannover, Hong Kong, St. Petersburg and Brazil. His main interests are the representation and processing of uncertainty, especially interval computations and intelligent control. He has published seven books, twenty edited books, and more than 1,400 papers. Vladik is a member of the editorial board of the international journal "Reliable Computing" and several other journals. In addition, he is the co-maintainer of the international Web site on interval computations http://www.cs.utep.edu/interval-comp. Vladik is Vice President for Publications of IEEE Systems, Man, and Cybernetics Society, Vice President for Publicity of the International Fuzzy Systems Association (IFSA), Vice President of the European Society for Fuzzy Logic and Technology (EUSFLAT), Fellow of International Fuzzy Systems Association (IFSA), Fellow of Mexican Society for Artificial Intelligence (SMIA).